ANALYSIS OF AIRCRAFT TECHNICAL OPERATION PROCESSES USING MARKOV PROCESSES AND CHAINS

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Abstract: *A technical operation of the aircraft and its avionics is investigated as a process with the successions of states of operation. The models of the processes and systems of technical operations of avionics in the form of stationary and nonstationary Markov chains of the first and higher order are proposed. There are examples of the processes for two states of operation – able to work and disabled states.*

АНАЛИЗ НА ПРОЦЕСИТЕ НА ТЕХНИЧЕСКА ЕКСПЛОАТАЦИЯ НА ВЪЗДУХОПЛАВАТЕЛНИ СРЕДСТВА С ИЗПОЛЗВАНЕ НА ПРОЦЕСИ И ВЕРИГИ НА МАРКОВ

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Резюме: *Техническата експлоатация на въздухоплавателните средства и тяхната авионика се разглежда като процес, включващ последователност от експлоатационни състояния. Предложени са модели на процесите и системите за техническа експлоатация на авиониката под формата на стационарни и нестационарни вериги на Марков от първи и по-висок ред. Разгледани са примери за процеси с две експлоатационни състояния – състояние на изправност и състояние на неизправност.*

Introduction

The operation of any aircraft involves the constant impact of various factors on its components, leading to random or systematic changes in their technical condition. Markov processes are a specific type of stochastic process occurring during aircraft operation. The clear link between the reliability characteristics of these systems and the costs associated with their restoration enables the application of Markov process analysis to the management of aircraft operational maintainability. Managing operational maintainability using Markov chains involves determining the optimal maintenance and repair strategy (or course of action) for each state of the asset, thereby minimizing operational costs. This is particularly relevant to the avionics systems of 4+ and 4++ generation aircraft and helicopters.

Models of Technical Operation Processes and Systems

In a broad sense, avionics can be understood as a field of engineering science concerned with the development, manufacture, and operation of aviation electronic and automatic equipment.

In a narrow sense, avionics can be understood as on-board electronic equipment that is, any electronic device (including its electrical components) intended for use on board an aircraft, such as radio-electronic equipment, automatic flight control systems, and instrumentation. This definition of avionics is provided in ICAO manuals [1]. An analysis of the structure of modern avionics systems

and systems in civil aviation aircraft is presented in [2]. The fundamentals of the theory regarding the technical operation of flight and navigation equipment (FNE) – an integral part of avionics – are described in [3].

The system for the technical operation of aircraft avionics comprises the technical assets and equipment, maintenance and repair programs, and the personnel who execute procedures and organize the processes involved in the technical operation of the avionics.

The quality of the technical operation system manifests itself at all stages of the technical operation process and comprises a complex of processes involving intended use, operational monitoring, maintenance, restoration, and repair.

The technical operation of an aircraft and its avionics can be viewed as a process involving sequential changes in operational states. During intended use, these states can be categorized as functional, serviceable, unserviceable, and non-functional. Monitoring avionics operational states encompasses pre-flight and post-flight checks, in-flight monitoring, and the assessment of removed equipment to determine whether components are operational, serviceable, non-functional, or defective. Similarly, distinctions can be made regarding operational and periodic maintenance states, as well as corrective and preventive restoration and repair. When analyzing these processes in real time, it is necessary to take into account states of readiness, downtime, and storage.

The maintenance process viewed as a sequence of changing maintenance states unfolds over time under the influence of numerous factors, both random and non-random. These include the occurrence of failures and malfunctions, the quality of maintenance and repair, the reliability of monitoring equipment, the availability of spare parts, and so on. Transitions between maintenance states occur at both random and deterministic points in time. Such transitions are driven by event streams (e.g., streams of failures, repairs, inspection procedures, etc.). Thus, the maintenance process can be modeled as a stochastic process defined over a set of operational states, characterized by transition probabilities. This enables the analysis of avionics processes and systems by leveraging the well-established framework of stochastic process theory.

By its very nature, the avionics process is a controlled one. The potential for intervention in this process is both objectively deterministic governed by the operating manual and the avionics maintenance and repair program and subjectively random, resulting from unauthorized or erroneous actions by flight and engineering personnel.

A Markov chain is a discrete-time stochastic process characterized by the complete or partial independence of its transitions from the past. The simplest such chain is a first-order stationary Markov chain, in which transitions are independent of both past transitions and time. Another type is the first-order non-stationary Markov chain, where transitions are independent of past transitions but depend on time. Higher-order chains whose transitions depend partially or entirely on previous ones represent a generalization of stationary Markov chains. The most complex objects in Markov chain theory are non-stationary higher-order chains, where transitions depend on both previous transitions and the specific points in time at which those transitions occurred. Thus, four basic types of Markov chains are defined.

Markov chains as a model of technical operation processes

How can technical operation processes be defined as Markov chains? The description of Markov chains begins with defining the set of states, $\boldsymbol{S}$. In the general case, this set may be countable. Here, however, we will limit ourselves to considering a set $\boldsymbol{S}$ consisting of $\boldsymbol{n}$ states:

(1) $S = \{S_1, S_2, \dots, S_i, \dots, S_j, \dots, S_n\}$.

At the initial time instant $[l = 0]$, the probability distribution of the states must be specified:

(2) $P^{(0)} = \left\{P_1^{(0)}, P_2^{(0)}, \dots, P_i^{(0)}, \dots, P_j^{(0)}, \dots, P_n^{(0)}\right\}$,

provided it is valid $\sum_{i=1}^{n} P_i^{(0)} = 1$.

The superscript index in parentheses will denote time instants. In the special case, $P^{(0)}$ represents a degenerate unit vector - that is, a vector that ought to be a unit vector but is a zero vector and cannot be normalized:

(3) $P^{(0)} = \{0,0, \dots, l, \dots,0\}$.

If, at a given time l, the chain is in state $S_i \in S$, then the next change (value) of the stationary Markov chain is determined by the transition probability matrix

(4) $P_l^S = \{p(S_j/S_i), S_j, S_i \in S\}$,

whose elements depend solely on the current state of S_i and do not depend on previous changes or the time instant l. Thus, a stationary Markov chain of order l is defined, representing the set

(5) $\xi_1^S = \{S, P^{(0)}, P_l^S\}$.

The dynamics of stationary Markov chains of the type ξ_l^S are described as follows:

(6) $h_{\xi_1^S}(m) = S_{i_0}^{(0)}, S_{i_1}^{(1)}, \dots, S_{i_m}^{(m)}$.

The step number of the Markov chain serves as a time parameter. The expression $S_{i_m}^{(m)}$ denotes the state of the chain $S_{i_0} \epsilon S$ at time m. The realization of the stationary Markov chain $h_{\xi_1^S}(m)$ corresponds to the probability:

(7) $P\left\{h_{\xi_l^S}(m)\right\} = P_{i_0}^{(0)} p\left(S_{i_1}^{(1)}/S_{i_0}^{(0)}\right) \dots p\left(S_{i_m}^{(m)}/S_{i_{m-1}}^{(m-1)}\right)$.

Let us denote by $p^{(m)}\left(S_j/S_i\right)$ the conditional probability that the stationary Markov chain enters state S_j at the m-th step, given that the initial state was S_i. It is equal to the sum of the probabilities of all realizations of length m that start at S_i and end at S_j. Specifically, $p^{(1)}\left(S_j/S_i\right) = p\left(S_j/S_i\right)$:

(8) $p^{(2)}\left(S_j/S_i\right) = \sum_{k=1}^{n} p(S_k/S_i) p\left(S_j/S_k\right)$.

By induction, the general Kolmogorov-Chapman recurrence formula for homogeneous Markov stochastic processes is obtained:

(9) $p^{(m)}\left(S_j/S_i\right) = \sum_{k=1}^{n} p^{(l)}(S_k/S_i)\, p^{(m-1)}\left(S_j/S_i\right)$.

Equation (9) reflects the fact that the first l steps bring the stationary Markov chain to state S_k, and the probability of the subsequent transition from S_k to S_j does not depend on the way the process reached state S_k.

It follows from equation (9) that the unconditional probability of reaching state S_j at step m can be determined using the following recurrence formula:

(10) $P^{(m)}\left(S_j\right) = \sum_{k=1}^{n} P^{(0)}\,(S_k) p^{(m)}\left(S_j/S_k\right)$.

State S_j is reachable from state S_i if there exists a value $m \geq 0$ such that $p^{(m)}\left(S_j/S_k\right) > 0$ – that is, if there is a positive probability of transitioning from state S_i to state S_j. A Markov chain is irreducible when every state is reachable from every other state. Let $f^{(m)}\left(S_j/S_k\right)$ denote the probability that, starting from state S_i, the first occurrence of state S_j takes place after m steps. If we set $f^{(0)}\left(S_j/S_i\right) = 0$, then

(11) $f^{(\infty)}\left(S_j/S_i\right) = \sum_{l=1}^{\infty} f^{(l)}\left(S_j/S_i\right)$.

The probability $f^{(\infty)}\left(S_j/S_i\right)$ represents the probability that, starting from state S_i, the process will at some point transition to state S_j.

To calculate the probability $f^{(\infty)}\left(S_j/S_i\right)$, the following reasoning is applied. If the first arrival at state S_j occurs at transition l, where $(1 \leq l \leq m-1)$, then the conditional probability of reaching state S_j at transition m will be equal to $p^{(m)}\left(S_j/S_i\right)$. Taking into account that $p^{(0)}\left(S_j/S_i\right) = 1$, we obtain

(12) $p^{(m)}\left(S_j/S_i\right) = \sum_{l=1}^{m} f^{(1)}\left(S_j/S_i\right) p^{(m-1)}\left(S_j/S_i\right)$.

In an iterative cycle, by successively assigning values to the transition indices $l = 1,2,\dots,n$, values for $p^{(l)}\left(S_j/S_i\right)$ are obtained via formula (12); when substituted into (11), these allow for the determination of results for $f^{l}\left(S_j/S_i\right)$ for values of $l = \overline{1,m}$.

Then, the mean return time of the process $\mu\left(S_j\right)$ to state S_j can be determined by the following expression:

(13) $\mu\left(S_j\right) = \sum_{l=1}^{\infty} l f^{(l)}\left(S_j/S_i\right)$.

A state S_i is recurrent if $f^{(\infty)}\left(S_j/S_i\right) = 1$, and is non-recurrent if $f^{(\infty)}\left(S_j/S_i\right) < 1$. A state S_j has period $t > 1$, if $p^{(m)}\left(S_j/S_i\right) = 0$, where m is not a multiple of t and t is the largest integer with this property. A state S_j is non-periodic if no such $t > 1$ exists. A non-periodic recurrent state S_j for which $\mu\left(S_j\right) < \infty$ is an ergodic state.

In an irreducible ergodic chain, there exist limits that do not depend on the initial state S_j. These limits are defined as:

(14) $\pi\left(S_j\right) = lim\, p^{(m)}\left(S_j/S_i\right)$,

as $\pi\left(S_j\right) > 0$ and

(15) $\pi\left(S_j\right) = \sum_{i=1}^{n} \pi(S_i)\, p\,\left(S_j/S_i\right)$, for $S_j \in S$,

where $\sum_{i=1}^{n} \pi(S_i) = 1$ and $\mu\left(S_j\right) = 1/\pi\left(S_j\right)$.

For a non-stationary Markov chain, the transition probability matrix takes the following form:

(16) $\mathfrak{R}_l^{NS} = \{P_l^{NS}, l = \overline{0,m}\}$,

where $P_l^{NS} = \{p(S_j/S_i, l), S_i, S_j \in S\}$ are matrices whose elements depend on the time instant l.

Thus, the non-stationary Markov chain of order 1 is written as

(17) $\xi_l^{NS} = \{S, P^{(0)}, \mathfrak{R}_l^{NS}\}$.

The flow of processes in stationary Markov chains of the form $h_{\xi_l^S}(m)$ corresponds to a probability

(18) $P\{h_{\xi_l^{NS}}(m)\} = P_{i_0}^{(0)} p(S_{i_1}^{(1)}/S_{i_0}^{(0)}, 0) \dots p(S_{i_m}^{(m)}/S_{i_{m-1}}^{(m-1)}, m-1)$.

The conditional probability $p^{(m)}(S_j/S_i, m)$ for the non-stationary Markov chain to fall into state S_j at the next m step, given that the initial state of the process was S_i, is equal to the sum of the probabilities of all realized states for the time period m, starting from state S_i and ending in state S_j:

(19) $p^{(1)}(S_j/S_i, 1) = p(S_j/S_i, 1)$;
$p^{(2)}(S_j/S_i, 2) = \sum_{i=1}^n p^{(1)}(S_k/S_i, 1)\, p(S_j/S_k, 2)$.

Then, by induction, the general Kolmogorov-Chapman recurrence formula for a nonstationary Markov chain is derived:

(20) $p^{(m)}(S_j/S_i, m) = \sum_{k=1}^n p^{(1)}(S_k/S_i, 1)\, p^{(m-1)}(S_j/S_k, m-1)$.

The unconditional probability of reaching the non-stationary Markov process in state S_j after executing m steps is determined as follows:

(21) $p^{(m)}(S_j, m) = \sum_{k=1}^n p^{(0)}(S_k, 1)\, p^{(m)}(S_j/S_k, m)$.

If $f^{(m)}(S_j/S_i, m)$ denotes the probability that starting from the initial state S_i the non-stationary Markov process will reach state S_j after exactly m number of steps. To calculate this probability, by analogy with stationary Markov chains, formula (12) can be used, and we will obtain:

(22) $p^{(m)}(S_j/S_i, m) = \sum_{k=1}^n f^{(1)}(S_j/S_i, 1)\, p^{(m-1)}(S_j/S_k, m-1)$.

Then the average time for the non-stationary process $\mu_{\mathfrak{R}_l^{NS}}(S_j)$ to return to state S_j can be determined as follows:

(23) $\mu_{\mathfrak{R}_l^{NS}}(S_j) = \sum_{l=1}^{\infty} l f^{(l)}(S_j/S_i, l)$.

Consider a stationary Markov chain of order k. Such a chain is defined by the following set:

(24) $\xi_k^S = \{S, P^{(0)}, \mathfrak{R}_k^S\}$,

where $\mathfrak{R}_k^S = \{P_l^S, l = \overline{1,k}\}$ is the set of transition state matrices from row l to row k. The elements of these matrices are defined as follows:

(25) $P_l^S = \{p(S_{i_l}/S_{i_{l-1}}), S_{i_l}, S_{i_{l-1}}, S_{i_0} \in S\}$

and depend on the previous l states.

The realization of $h_{\xi_k^S}(m)$ corresponds to the probability

(26) $P\{h_{\xi_k^S}(m)\} = P_{i_0}^{(1)} p(S_{i_1}^{(1)}/S_{i_0}^{(0)}) \dots p(S_{i_k}^{(k)}/S_{i_{k-1}}^{(k-1)}, \dots, S_{i_0}^{(0)}) \dots p(S_{i_m}^{(m)}/S_{i_{m-1}}^{(m-1)}, \dots, S_{i_{m-k}}^{(m-k)})$.

Let $p^{(m)}(S_{i_m}/S_{i_{m-1}}, \dots, S_{i_{m-k}})$ denote the conditional probability of a Stationary Markov Chain entering state S_{i_m} after m-number of steps, if the initial state at time instant $(m-k)$ was $S_{i_{m-k}}$. Then we can write:

(27)
$p^{(1)}(S_j/S_i) = p(S_{i_1}/S_{i_0})$;
$p^{(2)}(S_{i_2}/S_{i_1}, S_{i_0}) = \sum_{i_1=1}^n p^{(1)}(S_{i_1}/S_{i_0}) p(S_{i_2}/S_{i_1}, S_{i_0})$;
...
$p^{(k)}(S_{i_k}/S_{i_{k-1}}, \dots, S_{i_0}) = \sum_{i_{k-1}=1}^n p^{(k-1)}(S_{i_k}/S_{i_{k-1}}, \dots, S_{i_0}) p(S_{i_k}/S_{i_{k-1}}, \dots, S_{i_0})$;
...
$p^{(m)}(S_{i_m}/S_{i_{m-1}}, \dots, S_{i_{m-k}}) = \sum_{i_{m-1}=1}^n p^{(m-1)}(S_{i_{m-1}}/S_{i_{m-2}}, \dots, S_{i_{m-k}}) p(S_{i_{m-1}}/S_{i_{m-2}}, \dots, S_{i_{m-k}})$.

If $m = k$, a non-stationary Markov chain will be obtained, the transitions in which will depend on all previous transitions.

From (27) it follows that the unconditional probability of entering state S_{i_m} at m consecutive steps with number m can be determined using the recurrence formula:

(28) $p^{(m)}(S_{i_m}) = \sum_{i_0=1}^n P^{(0)}(S_{i_0}) p^{(m)}(S_{i_m}/S_{i_{m-1}}, \dots, S_{i_0})$.

If from an initial state S_{i_0} we express with $f^{(m)}\left(S_{i_m}/S_{i_{m-1}},\dots,S_{i_0}\right)$ the probability of first entering state S_{i_m} at step m. The expression is valid for this probability:

(29) $f^{(m)}\left(S_{i_m}/S_{i_{m-1}},\dots,S_{i_0}\right)=\sum_{l=1}^{n}f^{(l)}\left(S_{i_m}/S_{i_{m-1}},\dots,S_{i_0}\right)p^{(m-l)}\left(S_{i_m}/S_{i_{m-1}},\dots,S_{i_0}\right)$.

Then the average return time of $\mu_{\xi_l^{NS}}(S_i)$ of the high-order stationary Markov chain to state S_i is determined as follows:

(30) $\mu_{\xi_l^{NS}}(S_i)=\sum_{l=1}^{\infty}lf^{(l)}\left(S_{i_l}/S_{i_{l-1}},\dots,S_{i_{l-k}}\right)$.

Next, let's consider a non-stationary Markov chain of order k. It is defined by the following set:

(31) $\xi_k^{NS}=\left\{S,P^{(0)},\Re_k^{NS}(l)\right\}$,

where $\Re_k^{NS}=\left\{P_r^{NS}(l),\ l=\overline{1,m},\ r=\overline{1,k}\right\}$ is a set of transition probability matrices from l-th to k-th order, the elements of which depend on the previous k states and on the time instant l:

(32) $P_r^{NS}(l)=\left\{p\left(S_{i_k}/S_{i_{k-1}},l_{k-1},\dots,S_{i_0},l_0\right),S_{i_k},S_{i_{k-1}},\dots,S_{i_0}\in S\right\}$.

The realization $h_{\xi_k^{NS}}(m)$ of a high-order nonstationary Markov chain corresponds to a probability

(33) $$P\left\{h_{\xi_k^{NS}}(m)\right\}=P_{i_0}^{(0)}p\left(S_{i_1}/S_{i_0},0\right)\dots p\left[S_{i_k}^{(k)}/S_{i_{k-1}}^{(k-1)},(k-1),\dots,S_{i_0}^{(0)},0\right]\dots$$
$$\dots p\left[S_{i_m}^{(m)}/S_{i_{m-1}}^{(m-1)},(m-1),\dots,S_{i_{m-k}}^{(m-k)},(m-k)\right].$$

In this way, the main types of Markov chains were examined as models of the processes of technical operation of aircraft avionics.

Stationary Markov chain model of the technical operation process

The determination of the analytical functions of the failure intensity and the probability of failure-free operation of avionics based on statistical data is shown in work [3]. As is known, objects of technical operation in aviation can be in two states: S_1 – operational state for use as intended, and S_2 – inoperative state, in which the operational state is restored.

At time $l=0$ the object is in state S_2, i.e. $P^{(0)}=(0,1)$. The transition probability matrix is written in the following form:

$P_l^S=\begin{bmatrix}0{,}99 & 0{,}01\\ 1 & 0\end{bmatrix}$.

The probability of the object falling into states S_1 and S_2 in four steps can be determined as follows:

$p^{(1)}(S_1/S_2)=1;$
$p^{(1)}(S_2/S_2)=0;$
$p^{(2)}(S_1/S_2)=p^{(1)}(S_1/S_2)p(S_1/S_2)+p^{(1)}(S_2/S_2)p(S_1/S_2)=0{,}99;$
$p^{(2)}(S_2/S_2)=p^{(1)}(S_1/S_2)p(S_1/S_2)+p^{(1)}(S_2/S_2)p(S_2/S_2)=0{,}01;$
$p^{(3)}(S_1/S_2)=p^{(2)}(S_1/S_2)p(S_1/S_2)+p^{(2)}(S_2/S_2)p(S_1/S_2)=0{,}9901;$
$p^{(3)}(S_2/S_2)=p^{(2)}(S_1/S_2)p(S_1/S_2)+p^{(2)}(S_2/S_2)p(S_2/S_2)=0{,}0099;$
$p^{(4)}(S_1/S_2)=p^{(3)}(S_1/S_2)p(S_1/S_2)+p^{(3)}(S_2/S_2)p(S_1/S_2)=0{,}990099;$
$p^{(4)}(S_2/S_2)=p^{(3)}(S_1/S_2)p(S_1/S_2)+p^{(3)}(S_2/S_2)p(S_2/S_2)=0{,}009901.$

The unconditional probability of the object falling into states S_1 and S_2 in four steps can be determined as follows:

$p^{(1)}(S_1)=p^{(0)}(S_1)p(S_1/S_1)+p^{(0)}(S_2)p(S_1/S_2)=1;$
$p^{(1)}(S_2)=0;$
$p^{(2)}(S_1)=p^{(1)}(S_1)p(S_1/S_1)+p^{(1)}(S_2)p(S_1/S_2)=0{,}99;$
$p^{(2)}(S_2)=0{,}01;$
$p^{(3)}(S_1)=p^{(2)}(S_1)p(S_1/S_1)+p^{(2)}(S_2)p(S_1/S_2)=0{,}9901;$
$p^{(3)}(S_2)=0{,}0099;$
$p^{(4)}(S_1)=p^{(3)}(S_1)p(S_1/S_1)+p^{(3)}(S_2)p(S_1/S_2)=0{,}990099;$
$p^{(4)}(S_2)=0{,}009901.$

The probability of an event in which, starting from the initial state S_2, the object will first enter state S_2 between the first and fourth steps is determined as follows:

$f^{(1)}(S_2/S_2)=\frac{p^{(1)}(S_2/S_2)}{p^{(0)}(S_2/S_2)}=0;$
$f^{(2)}(S_2/S_2)=\frac{p^{(2)}(S_2/S_2)-f^{(1)}(S_2/S_2)p^{(1)}(S_2/S_2)}{p^{(0)}(S_2/S_2)}=0{,}01;$
$f^{(3)}(S_2/S_2)=\frac{\{p^{(3)}(S_2/S_2)-f^{(1)}(S_2/S_2)p^{(2)}(S_2/S_2)-f^{(2)}(S_2/S_2)p^{(1)}(S_2/S_2)\}}{p^{(0)}(S_2/S_2)}=0{,}0099;$
$f^{(4)}(S_2/S_2)=\frac{\{p^{(4)}(S_2/S_2)-f^{(1)}(S_2/S_2)p^{(3)}(S_2/S_2)-f^{(2)}(S_2/S_2)p^{(2)}(S_2/S_2)-f^{(3)}(S_2/S_2)p^{(1)}(S_2/S_2)\}}{p^{(0)}(S_2/S_2)}=0{,}009801.$

The average time for the object to return to state S_2 is determined by formula (13):

$$\mu(S_2) = 1.f^{(1)}(S_2/S_2) + 2.f^{(2)}(S_2/S_2) + 3.f^{(3)}(S_2/S_2) + 4.f^{(1)}(S_2/S_2) =$$

$$= 1.2 + 2.0{,}01 + 3.0{,}0099 + 4.0{,}009801 = 2.0{,}01 + \sum_{l=3}^{\infty} 0{,}99^{(l-1)}\, 0{,}01.$$

The stationary probabilities for states S_1 and S_2 can be determined by formulas (14) and (15):

$\pi(S_1) = \pi(S_1)p(S_1/S_1) + \pi(S_2)p(S_1/S_2)$,

$\pi(S_1) + \pi(S_2) = 1$,

$\pi(S_1) = \pi(S_1)p(S_1/S_1) + [1 - \pi(S_1)]p(S_1/S_2)$,

$\pi(S_1) = \frac{p(S_1/S_1)}{1-p(S_1/S_1)+p(S_1/S_2)} = \frac{1}{1-0{,}99+1} = \frac{1}{1{,}01}0{,}99(0099)$;

$\pi(S_2) = 0{,}00(9901)$.

The average time $\mu(S_2)$ can be determined as follows:

$\mu(S_2) = \frac{1}{\pi(S_2)} = \frac{1}{0{,}00(9901)} \approx 100{,}91$.

Non-stationary Markov chain model of the technical operation process

Let us assume the following form of the transition probability matrix:

$p_1^{NS} = \begin{pmatrix} 0{,}99 & 0{,}01 \\ 1 & 0 \end{pmatrix}$; $\quad p_2^{NS} = \begin{pmatrix} 0{,}98 & 0{,}02 \\ 1 & 0 \end{pmatrix}$;

$p_3^{NS} = \begin{pmatrix} 0{,}97 & 0{,}03 \\ 1 & 0 \end{pmatrix}$; $\quad p_4^{NS} = \begin{pmatrix} 0{,}96 & 0{,}04 \\ 1 & 0 \end{pmatrix}$.

This shows that the states of the object are described by a non-stationary Markov chain.

The probability of the object falling into states S_1 and S_2 in four steps can be determined as follows:

$p^{(1)}(S_1/S_2\,,1) = 1$;

$p^{(1)}(S_2/S_2\,,1) = 0$;

$p^{(2)}(S_1/S_2\,,2) = p^{(1)}(S_1/S_2\,,1)p(S_1/S_2\,,2) + p^{(1)}(S_2/S_2\,,1) + p(S_1/S_2\,,2) = 0{,}98$.

$p^{(2)}(S_2/S_2\,,2) = 1 - 0{,}98 = 0{,}02$;

$p^{(3)}(S_1/S_2\,,3) = p^{(2)}(S_2/S_2\,,2)p(S_1/S_1\,,3) + p^{(2)}(S_2/S_2\,,2)p(S_1/S_2\,,3) = 0{,}98.0{,}97 + 0{,}2.1 =$

$= 0{,}9706$;

$p^{(3)}(S_2/S_2\,,3) = 1 - 0{,}9706 = 0{,}0294$;

$p^{(4)}(S_1/S_2\,,4) = p^{(3)}(S_2/S_2\,,3)p(S_1/S_1\,,4) + p^{(3)}(S_2/S_2\,,3)p(S_1/S_2\,,4) =$

$= 0{,}9706.0{,}96 + 0{,}0294.1 = 0{,}991176$.

$p^{(4)}(S_2/S_2\,,4) = 1 - p^{(4)}(S_1/S_2\,,4)$.

The probability of an event in which, starting from the initial state S_1, the object will first enter state S_2 between the first and fourth steps is determined as follows:

$f^{(1)}(S_2/S_2\,,1) = \frac{p^{(1)}(S_2/S_2,1)}{p^{(0)}(S_2/S_2,0)} = 0$;

$f^{(2)}(S_2/S_2\,,2) = \frac{p^{(2)}(S_2/S_2,2) - f^{(1)}(S_2/S_2,1)p^{(1)}(S_2/S_2,1)}{p^{(0)}(S_2/S_2,0)} = 0{,}02$;

$f^{(3)}(S_2/S_2\,,3) = \frac{\{p^{(3)}(S_2/S_2,3) - f^{(1)}(S_2/S_2,1)p^{(2)}(S_2/S_2,2) - f^{(2)}(S_2/S_2,2)p^{(1)}(S_2/S_2,1)\}}{p^{(0)}(S_2/S_2,0)} = 0{,}0294$;

$f^{(4)}(S_2/S_2\,,4) = \frac{\{p^{(4)}(S_2/S_2,4) - f^{(1)}(S_2/S_2,1)p^{(3)}(S_2/S_2,3) - f^{(2)}(S_2/S_2,2)p^{(2)}(S_2/S_2,2) - f^{(3)}(S_2/S_2,3)p^{(1)}(S_2/S_2,1)\}}{p^{(0)}(S_2/S_2,0)} =$

$= 0{,}0306$.

The average time for the object to return to state S_2 can be determined by formula (23). In this way, the principle possibility of describing the simplest processes of technical operation of avionics of aviation equipment using Markov processes and chains is shown.

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